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Do principal surfaces of stress and strain always exist?

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Abstract—Variations of stress and strain are commonly expressed by patterns of stress or strain trajectories: three mutually orthogonal families of continuous lines, parallel to maximum, intermediate and minimum stress or strain axes. It might be assumed that there are equivalent continuous *principal surfaces* of stress or strain, for any state of continuously varying stress or strain. We demonstrate that this will not generally be the case for three-dimensionally varying states of stress or strain.

Whether or not principal surfaces of stress or strain exist is governed by the *abnormality of the vector field* of principal trajectories. We consider the Z vector fields for examples of many types of three-dimensional heterogeneous deformation, and show that most of these do not lead to definable principal XY strain surfaces. An alternative geometric test is presented, termed the *continuity loop*, for simply demonstrating the existence (or not) of principal surfaces, using geometrical and orientational information.

It is important to the understanding of geological structures to know which kinds of heterogeneous deformation give rise to principal surfaces of stress or strain. We conclude with examples of structures which might be indicative of the absence of continuous principal surfaces of stress (segmented faults, echelon veins and dykes), a discussion of the implication for strain fabrics and foliations, and a warning that foliation trace trajectories on maps or sections may not necessarily indicate the existence of real foliation surfaces in three dimensions. © 1997 Elsevier Science Ltd.

INTRODUCTION

The concepts of *principal planes* of stress and strain are important in structural geology, and figure in virtually all current text books in connection with fracture and fabric forming processes. Principal planes of stress are considered to control the orientations of brittle structures and their associations: faults, joints, mineral veins, dykes and other igneous intrusions. Principal planes of strain are assumed to control ductile fabrics in rocks, with cleavages and foliations widely believed to form parallel to the XY principal plane (denoting strain ellipsoid axes, $X \geq Y \geq Z$).

In states of locally or regionally varying stress or strain, indicated by curving stress or strain trajectories, principal planes can only be defined at a point. On a larger scale, they would be described as *principal surfaces* with expected curving form. It might be automatically assumed that for *any* family of continuously curving stress trajectories (σ_1, σ_2, σ_3; compressive stress positive, with $\sigma_1 \geq \sigma_2 \geq \sigma_3$) there is a related family of continuous curved principal $\sigma_1\sigma_2$, $\sigma_2\sigma_3$ and $\sigma_1\sigma_3$ surfaces. Likewise, a set of curving X, Y and Z strain trajectories might be assumed to have three associated curving XY, YZ and XZ surfaces, mutually orthogonal. This paper will show that such general assumptions are not correct.

Mandl (1987) questioned the existence of continuous principal surfaces of stress, in connection with discontinuous faults. Through examination of vector fields for non-uniform and non-plane stress, he demonstrated that there were states of three-dimensional heterogeneous stress which could not possess continuous principal surfaces. It followed that the associated Coulomb–Mohr shear fractures would not be continuous surfaces either. Mandl (1987) therefore concluded that for general variations in states of stress (i.e. non-plane stress and non-plane strain), faulting should generally be discontinuous. He also noted that the same would be true of extensional fractures, for equivalent heterogeneous stress conditions. Mandl's examples include torsion of a circular cylinder, screw dislocations and a depth-varying tectonic stress field. The latter, shown in Fig. 1, is characterized by a constant vertical principal stress (here σ_1), and two horizontal principal stresses which rotate progressively with depth. A continuous principal $\sigma_1\sigma_2$ surface cannot be defined, as shown schematically by the separate 'flags' in Fig. 1(a). It follows that the Coulomb–Mohr fractures will not be continuous surfaces (Fig. 1b), and Mandl proposed this as a mechanism for segmented normal faults of the kind discussed by Segall and Pollard (1980).

Despite much current interest in fault geometry, including echelon and segmented faults (e.g. Peacock and Sanderson, 1991; Cartwright *et al.*, 1995; Childs *et al.*, 1996), Mandl's explanation that discontinuous faults might occur for states of stress which do not have

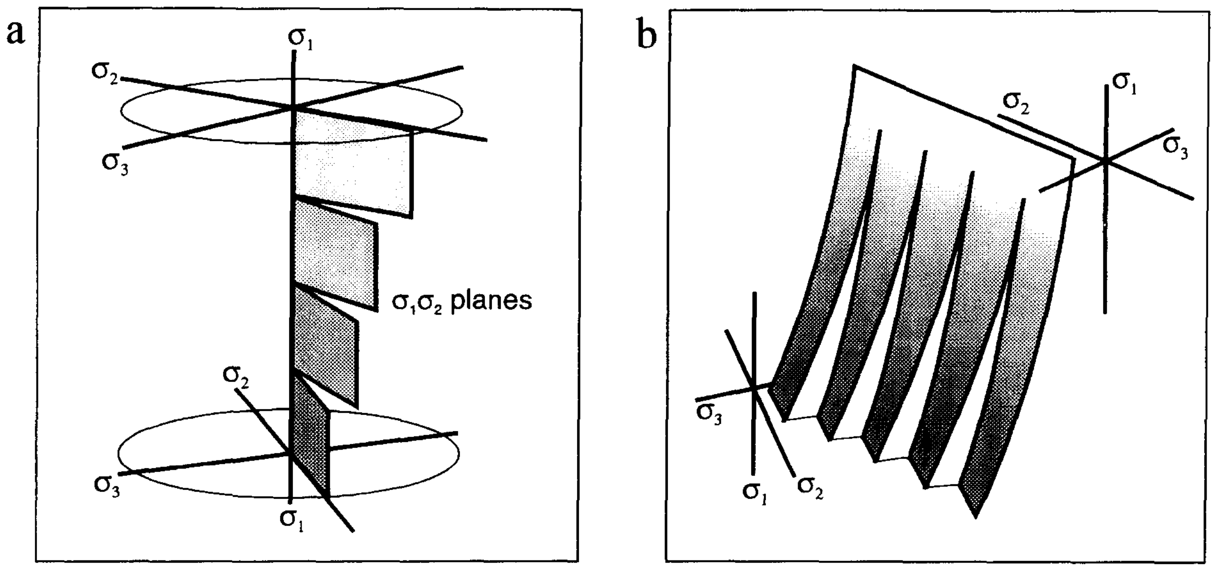

Fig. 1. Example of an undefinable principal surface of stress: the 'tectonic stress' state of Mandl (1987, figs 3 & 4). (a) The vertical stress, here σ_1, is constant, but the horizontal stresses progressively change and rotate with depth. A continuous $\sigma_1\sigma_2$ surface cannot be drawn. (b) Discontinuous faults (Coulomb–Mohr shear fractures) according to Mandl (1987), for the stress field in (a). The fault 'plane' becomes echelon segments, upwards or downwards.

continuous principal surfaces appears to have received little attention (Mandl, 1987). We have not found any specific attempts to prove or disprove Mandl's contention that shear fractures, tensile fractures and principal planes of stress must *generally* be discontinuous for non-uniform three-dimensional stress fields.

The question of whether principal surfaces of stress and strain exist for particular types of geological deformation appears not to have been asked widely in structural geology. It has been known for more than a century, in the mechanics literature (Boussinesq, 1872; Love, 1920, p. 87), that orthogonal surfaces to curving trajectories are the exception, not the rule. Yet structural geologists familiar with three-dimensional geometry and complexly curved surfaces, and educated to consider fractures and fabrics as surfaces, might find it harder to accept that principal surfaces of stress and strain do not necessarily exist. This point also raises problems of definition. Should a deformation where continuous principal surfaces do not exist (e.g. Fig. 1) be considered to possess a segmented 'discontinuous surface'? Or is no principal surface definable at all? The concept of a 'discontinuous surface' could be misleading, as it causes attention to focus on the nature of the discontinuities or 'jumps'. Instead, we wish to focus attention primarily on the fact that there may be *no definable surfaces* perpendicular to a family of curving trajectories, whether the definition is mathematical or geometrical.

Torsion is probably the simplest type of deformation where it can be demonstrated, purely from geometrical arguments, that principal stress and strain surfaces do not always exist. Torsion of a circular cylinder gives rise to cylindrical $\sigma_1\sigma_3$ surfaces (Fig. 2a). It might be imagined that the $\sigma_1\sigma_2$ and $\sigma_2\sigma_3$ 'surfaces' would be of continuous spiralling (helicoidal) form. This was shown by Mandl (1987) not to be the case. No continuous $\sigma_1\sigma_2$ and $\sigma_2\sigma_3$ surfaces exist, and this can be demonstrated geometrically (Fig. 2) as follows. For torsion of an upright cylinder, the $\sigma_1\sigma_2$ and $\sigma_2\sigma_3$ planes at any point must maintain their $\pm 45°$ dips, and make $\pm 45°$ traces (the σ_1 and σ_3 trajectories) on successively inward-nesting cylindrical $\sigma_1\sigma_3$ surfaces. The supposed $\sigma_1\sigma_2$ and $\sigma_2\sigma_3$ 'surfaces' must also contain the horizontal cylinder radii (σ_2), forming the the strike lines on the 'surface' (Fig. 2b); these need to radiate (upwards and downwards), while the dip,must remain constant, at 45°. This is physically impossible because a true continuous spiral or helicoid (such as a 'screw-thread' or a smoothed spiral staircase) only maintains its continuity by steepening its dip inwards, reaching vertical at the spiral-axis. Hence, for torsional deformation, two of the three principal planes of stress (and also strain) cannot be described as continuous surfaces.

The starting point of our paper is Mandl's claim that, for most cases of continuously varying three-dimensional stress or strain, there will not be continuous principal planes or surfaces (Mandl, 1987). This has significant implications for geological structures supposedly associated with principal planes of stress and strain. It also raises questions about geological interpretations and extrapolations from two to three dimensions. In this paper, we develop two tests which can be applied to verify the existence or not of principal surfaces of stress or strain. The first is a mathematical test and concerns the patterns of three-dimensional orientations of principal axes (i.e. their vector fields). The second concerns geometry, and uses the orientations of surfaces or their traces. We then consider examples of three-dimensionally heterogeneous states of stress or strain, to see which might possess principal surfaces, and

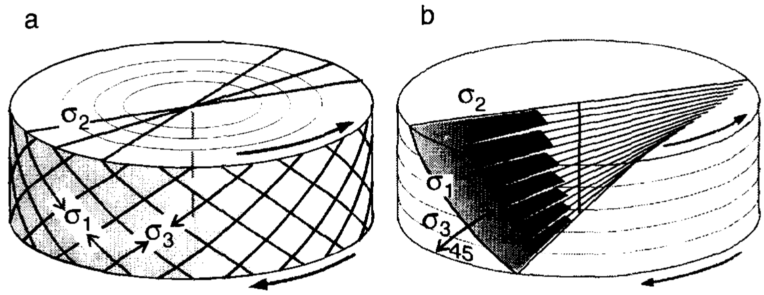

Fig. 2. The example of torsion of a circular cylinder. (a) Principal σ_1 and σ_3 trajectories wind around the cylinder, always maintaining angles of $\pm 45°$ with the torsion plane (horizontal), and σ_2 axes are always radial lines. (b) A continuous $\sigma_1\sigma_2$ surface (or $\sigma_2\sigma_3$ surface) cannot be drawn or defined, as shown by the 'broken' effect.

end with the implications for geological structures and fabrics.

PROOF OF THE EXISTENCE OF PRINCIPAL SURFACES, USING VECTOR FIELDS

In analyses of heterogeneous stress fields, surfaces which are everywhere orthogonal to a family of principal stress trajectories are known as the *principal stress surfaces* or *isostatic surfaces* (in older literature). Boussinesq (1872; see also Love, 1920, p. 87) pointed out that for three-dimensional stress patterns, the existence of isostatic surfaces is exceptional. From the point of view of structural geology, it is of interest to discover which deformations, and their configurations of stress or strain trajectories, can give rise to principal surfaces. This question was discussed by Björgum (1951), Erickson (1960) and Mandl (1987), and we will follow their analyses.

It is necessary to express principal trajectories as a *vector field*. The existence of surfaces which are orthogonal to a vector field is determined by an invariant property called the *abnormality* of the vector field. The abnormality, A, is defined as a scalar, which is derived from the dot product of the vector of the field $\mathbf{v}$ and another vector, the curl of that field, i.e.:

$$A = \mathbf{v} \cdot \operatorname{curl} \mathbf{v}. \qquad (1)$$

Only for zero abnormality, $A = 0$, expressed as

$$\mathbf{v} \cdot \operatorname{curl} \mathbf{v} = 0 \qquad (2)$$

is it possible to construct families of surfaces normal to the lines of the vector field $\mathbf{v}$.

The significance of curl $\mathbf{v}$ in equations (1) and (2) is best explained by considering $\mathbf{v}$ as a field of particle movement paths, rather than as a suite of stress or strain trajectories. In the context of this analogy, the curl describes the net rotation of the flow implied by the vector field. Curl $\mathbf{v}$ is the vector describing the axis and amount of the average rotation of all radii of a small sphere embedded in the flow (Fig. 3a). The abnormality, A, being the dot product of two vectors (equation 1), will be zero when the vectors of the field are normal to the curl; in other words, there will be no component of curl in the direction of $\mathbf{v}$. In these circumstances, the net rotation implied by neighbours to any vector line in the plane perpendicular to the latter will be zero (Fig. 3b). For this reason, the abnormality has been termed *the torsion of neighbouring vector lines* (Björgum, 1951). Although individual vector lines may possess torsion (i.e. depart from a plane curve), the family of immediate neighbours to a given vector line must not show a net 'coiling' about the latter, if the abnormality is zero. Referring to Fig. 3(b), the coiling of neighbouring vector lines is expressed by considering a small disc of radius r and normal to a vector line v. At points such as p on the circumference of the disc, a plane can be taken which is parallel to both the vector line v and the disc's tangent line. Within this plane, the angle θ is defined as the angle between the projections of v and the neighbour-

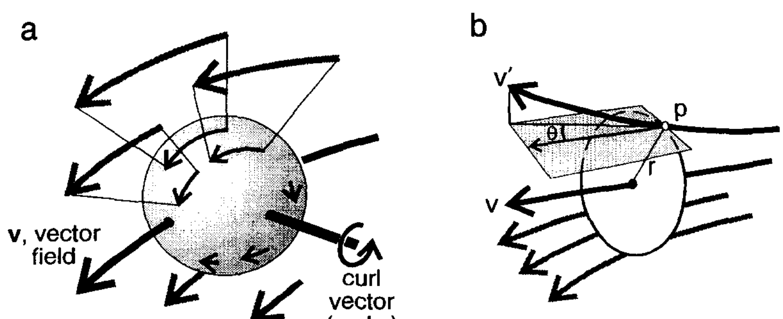

Fig. 3. (a) Illustration of the vector field, $\mathbf{v}$, and its curl. (b) The coiling of neighbouring vector lines can be considered in terms of a disc, radius r, normal to a vector line, v. See text for full discussion.

ing vector line v' which passes through p. If the net value of θ for all points around the rim is zero, the vector field has zero abnormality (Björgum, 1951, p. 26).

The essential property of a vector field controlling the existence of an orthogonal family of surfaces is that vectors are oriented at right angles to the curl, i.e. $A = 0$. This property is purely a feature of the geometry of the vector lines, and is independent of vector magnitudes. When considering whether or not the zero abnormality property exists for a given configuration of stress or strain trajectories, we can therefore replace the vector field, $\mathbf{v}$, with a directionally similar field, $\mathbf{t}$, made up of unit vectors.

If a family of orthogonal surfaces exists, the field of unit vectors obeys the equation

$$A = \mathbf{t} \cdot \mathrm{curl}\, \mathbf{t} = 0. \quad (3)$$

Written for a Cartesian x–y–z system,

$$\begin{aligned} A &= t_x \cdot (\mathrm{curl}\, \mathbf{t})_x + t_y \cdot (\mathrm{curl}\, \mathbf{t})_y + t_z \cdot (\mathrm{curl}\, \mathbf{t})_z \\ &\equiv t_x(\delta t_z/\delta y - \delta t_y/\delta z) \quad (4) \\ &\quad + t_y(\delta t_x/\delta z - \delta t_z/\delta x) + t_z(\delta t_y/\delta x - \delta t_x/\delta y). \end{aligned}$$

The conditions expressed in equations (3) and (4) will always be satisfied for a two-dimensional stress field in which σ_2 has a fixed orientation, as the directions of σ_1 and σ_3 will everywhere be contained in one plane. This is shown by considering the $\sigma_1\sigma_2$ surface, and hence the σ_3 unit vector, $\mathbf{t}$, taking the x–z plane to be the plane of stress. We have the following requirements for the vector components and derivatives for $\mathbf{t}$:

$$t_z = 0; \delta t_x/\delta z = \delta t_y/\delta z = 0. \quad (5)$$

Hence it is found that equation (3) is satisfied.

A vector field will thus always have orthogonal surfaces if there exists a coordinate system for which

$$t_x = f(x, y), t_y = g(x, y), t_z = 0. \quad (6)$$

The field is described as *plane* if equation (6) applies in some rectangular coordinate system, or *rotationally symmetric* (or axially symmetric), if equation (6) applies in cylindrical coordinates (Erickson, 1960, p. 824).

This vector field criterion for determining the existence of principal surfaces will be used for examples of heterogeneous deformation in a later section.

GEOMETRIC TESTS OF CONTINUITY OF PRINCIPAL SURFACES

Using orientational data: the continuity loop

The test of principal surface continuity in the previous section supposes that the vector fields for stress or strain trajectories can be defined. While it is possible to write expressions for vector fields for certain types of deformation (as will be given in the following section), the vector method may not be of much practical value for real geological data. Let us suppose that we have a series of strike and dip readings for a 'surface' which can be located in some geographical reference system: for example, a series of readings for a regional first cleavage on a map which includes topography. A simple geometrical method can test whether such data (e.g. for a regionally curving cleavage) can be characterized by a continuous curved surface in three dimensions, or not.

A geometrical property of all continuous surfaces (not just principal surfaces, as considered in this paper) can be described by *the continuity loop* (Fig. 4). In any three-dimensional block diagram, the traces of a continuous surface on the faces of the block will describe one or more continuous closed loops in three dimensions. We use a right-angled block, but the principle holds for any three-dimensional block shape. The type of loop(s) will depend on the surface curvature in relation to block size, ranging from a series of straight lines around the block for a planar surface (Fig. 4a), to several possible closed curved loops for doubly curving surfaces (e.g. Fig. 4b), to more complex shapes. The shape of the loops does not matter; the critical factor is that the starting and finishing points are the same (Fig. 4c), thus defining the continuity. Wherever the loop is not closed, and the starting and finishing points are not in the same place (e.g. Fig. 4d), a continuous surface cannot be defined.

The success of this simple graphical method depends on having accurate orientational data for strike, dip and position, or traces on sections which allow us to construct a block. The simplest test is a block aligned in the horizontal and vertical (Fig. 4), with edges N–S and E–W, so that the top face will always show the strike trace. Apparent dips on the vertical faces may be determined by stereographic construction, but the vertical positions of curved surfaces may be more difficult to resolve. However, it may be surprisingly easy to show that a supposed surface *cannot* be continuous for a series of readings. We will later use this to test for principal surface continuity in ductile transpression, as described by Robin and Cruden (1994).

Dupin's theorem

Where it can be shown from vector fields or continuity loops that continuous principal surfaces do exist, these are three families of mutually orthogonal principal surfaces (e.g. $\sigma_1\sigma_2$, $\sigma_2\sigma_3$, $\sigma_1\sigma_3$; or XY, YZ, XZ). What rules govern the geometry of these orthogonal principal surfaces? A theorem derived by the French mathematician C. Dupin in the early 19th century (see Nutbourne and Martin, 1988, p. 181) states: *If three families of surfaces meet orthogonally, the curves of intersection are lines of curvature on the surfaces*. Relating this theorem to the principal planes of strain, the three families of orthogonal surfaces represent the XY, YZ and XZ surfaces (Fig. 5). We know that their mutual intersections must be the X, Y and Z principal strain trajectories.

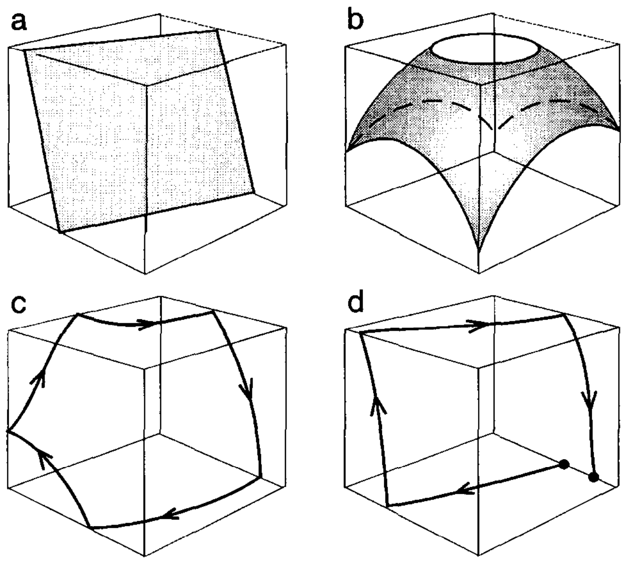

Fig. 4. *The continuity loop.* For a continuous surface, its intersections (i.e. traces) around any arbitrary block diagram will be continuous. (a) The 'loop' for a plane. (b) Loops for an arbitrary doubly curving surface. (c) Example of a continuous loop: start = finish. (d) A 'loop' which does not close, indicating a discontinuous surface: start ≠ finish: note gap between the two spots.

According to Dupin's theorem, these X, Y and Z trajectories must be *principal lines of curvature* on the principal surfaces. These lines of curvature can be called *principal curvature trajectories* (Fig. 5) because they track the local principal directions of curvature (Nutbourne and Martin, 1988, p. 125).

Dupin's theorem has important applications to principal surfaces of stress and strain. Lisle (oral communication, Tectonic Studies Group AGM 1994) proposed several uses of Dupin's theorem for the solution of structural problems. For example, if sets of foliations and lineations in a mapped area are valid indicators of principal strain surfaces and strain trajectories, respectively, then the geometries of these structures must obey Dupin's theorem. Thus, the direction of greatest stretching (X) must always be parallel to one set of principal curvature trajectories on the foliation surfaces, and Y parallel to the other set. The potential applications of this theorem may not be as broad as we first thought, however. To be applied, it must first be demonstrated that all three mutually orthogonal surfaces do indeed exist. As discussed above, and in the next section, only special arrangements of the strain trajectories lead to the existence of such surfaces. We now consider a range of examples of geologically realistic three-dimensional deformations.

EXAMPLES OF HETEROGENEOUS DEFORMATION IN THREE DIMENSIONS: THE EXISTENCE OR NOT OF PRINCIPAL SURFACES

Virtually all geological deformation can be described as heterogeneous, on one scale or another. Furthermore, geological structures are solid three-dimensional features. Should it be assumed, therefore, that strain is in general heterogeneous in three dimensions?

Most analytical and model studies simplify geological deformation to two dimensions, or consider the third dimension as uniform. This is understandable, first because the principles and mathematics become simpler, and second because the printed page forces us into two-dimensional views and representations. Examples of heterogeneous deformation in two dimensions (see Fig. 6) include strain across shear zones (Ramsay and Graham, 1970), fanning of strain around fold hinges (Dieterich, 1969) or around initially cylindrical inclusions (Shimamoto, 1975), and generalized modelling of strain gradients (e.g. Cutler and Elliott, 1983; Hirsinger and Hobbs, 1983; Cobbold and Barbotin, 1988). All these patterns of heterogeneous strain possess principal surfaces of strain because they are two dimensional, analogous to plane stress. The curving principal trajectory lines seen in the front planes of view in Fig. 6, and in successive parallel sections, are true traces of principal surfaces in three dimensions of cylindroidally curved form. They all have closed continuity loops.

Treatments of heterogeneous deformation in all three dimensions are rare in the geological literature. Despite the fact that many regions of natural structures exhibit regional swings of fold axes, cleavages and stretching lineations, a wholly three-dimensional approach is conceptually difficult. It is more usual to consider three-dimensional variation as successive *sections* across a region, providing successive traces of structures and fabrics. This can be done on the small scale, too, as serial sections or thin sections. These sections are not principal sections, however, and therefore two-dimensional theory and modelling are not truly applicable. In any general three-dimensionally heterogeneous deformation there is no single characteristic section which illustrates and characterizes the whole deformation.

We will consider some types of three-dimensionally heterogeneous deformation, and the presence or absence of principal surfaces, in two ways. (1) By the vector field method, we require expressions for the orientations of principal strain axes in an x–y–z coordinate system. (2) By the continuity loop method, we require orientational data on supposed principal surfaces so that a block diagram and traces can be drawn. Most of our examples use the first method.

Deformation induced by an inflating sphere or ballooning diapir

This deformation imposes a spherically symmetric principal compression (σ_1) or shortening (Z), which is independent of the orientation of any particular Cartesian coordinate frame. The treatments for stress and strain are equivalent. The case of rotational symmetry on the z coordinate axis has already been considered (equation (6), and it was shown that $A = 0$ and, therefore,

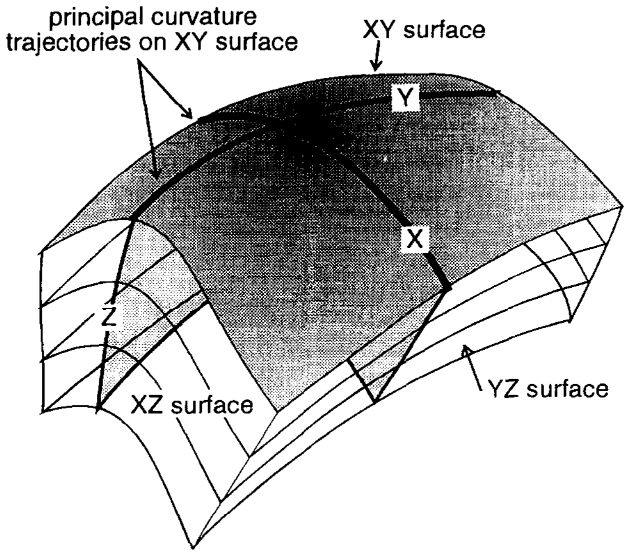

Fig. 5. Dupin's theorem: the three families of principal surfaces (XY, YZ, XZ) are orthogonal, and their mutual intersections (X, Y, Z trajectories) are lines of principal curvature.

that principal surfaces exist. Spherical symmetry, with interchangeability of x, y, z, is an even more special case than this, where

$$\delta t_x/\delta y = \delta t_y/\delta z = \delta t_z/\delta x = \delta t_z/\delta y, \text{ etc.,} \tag{7}$$

so (see equation (4) abnormality A is always 0. Consequently, XY principal surfaces (here with $X = Y$) will exist for this deformation.

Deformation induced by diapiric ascent of a sphere

A theoretical analysis of this problem (Schmeling *et al.*, 1988) treats flow as rotationally symmetric (axisymmetric) about the z axis, which thus makes the x and y coordinates and components interchangeable again. As stated above, it has been shown that rotationally symmetric deformations satisfy equation (2) and therefore possess principal surfaces. In this case, the XY surfaces might be conceived as onion-shaped or of conical form.

Simple shear in three dimensions

Simple shear sensu stricto is a two-dimensional plane-strain deformation, regardless of whether the shear is uniform or heterogeneous (Fig. 7a). It follows that this will give rise to principal surfaces (e.g. Figure 6b). Nevertheless, we will provide proof, using vector algebra, to lay the foundations for 'three-dimensional simple shear', treated as Cases A and B below.

For a simple shear in the x–z coordinate plane with

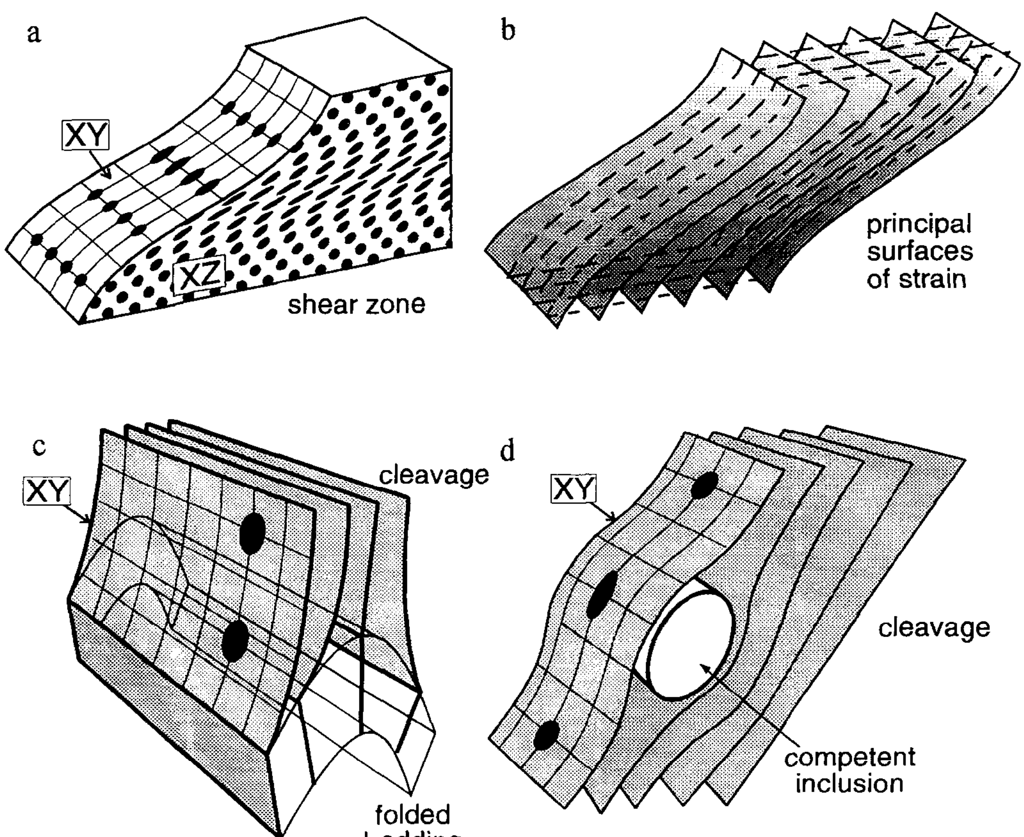

Fig. 6. Traditional examples of heterogeneous deformation. (a) Heterogeneous simple shear and (b) its principal XY surfaces. Strain variation and XY surfaces around (c) folds and (d) inclusions. All these deformations are two-dimensional (plane), and the XY surfaces have continuous cylindroidal form.

shear direction, x, the vector field of finite shortening directions, Z, can be expressed by the unit vector field, **t**, with components:

$$t_x = f_1(y),\ t_y = f_2(y),\ t_z = 0. \tag{8}$$

Recalling the curl **t** components (equation (4), we have

$$\begin{aligned}
[\text{curl } \mathbf{t}]_x &\equiv (\delta t_z/\delta y - \delta t_y/\delta z) = 0 \\
[\text{curl } \mathbf{t}]_y &\equiv (\delta t_x/\delta z - \delta t_z/\delta x) = 0 \\
[\text{curl } \mathbf{t}]_z &\equiv (\delta t_y/\delta x - \delta t_x/\delta y) = -\delta f_1(y)/\delta y.
\end{aligned} \tag{9}$$

The third term in the expression

$$A = t_x \cdot (\text{curl } \mathbf{t})_x + t_y \cdot (\text{curl } \mathbf{t})_y + t_z \cdot (\text{curl } \mathbf{t})_z$$

(equation 4) reduces to zero, because t_z is zero. Therefore $A = 0$, proving the presence of XY surfaces perpendicular to the Z vectors.

Case A: uniform shear with a changing shear direction. In physical terms, this is a shear zone which gives rise to X or Z strain axes inclined at constant angle to the (shear) coordinate plane (x–z), but the strain axes rotate and twist upwards and downwards in the y direction (Fig. 7b). The components of the unit vector, **t**, for the Z axes are:

$$t_x = f_1(y),\ t_y = k,\ t_z = f_3(y). \tag{10}$$

Curl **t** has the components (see equation (4) or equation (9):

$$\begin{aligned}
[\text{curl } \mathbf{t}]_x &= \delta f_3(y)/\delta y \\
[\text{curl } \mathbf{t}]_y &= 0 \\
[\text{curl } \mathbf{t}]_z &= -\delta f_1(y)/\delta y.
\end{aligned} \tag{11}$$

However, equation (4) now does not reduce to $A = 0$, but gives

$$A = f_1(y)[\delta f_3(y)/\delta y] + 0 - f_3(y)[\delta f_1(y)/\delta y]. \tag{12}$$

So, for this example, $A \neq 0$ and continuous XY surfaces cannot be defined. This could also be demonstrated quite simply, by the 'continuity loop' method.

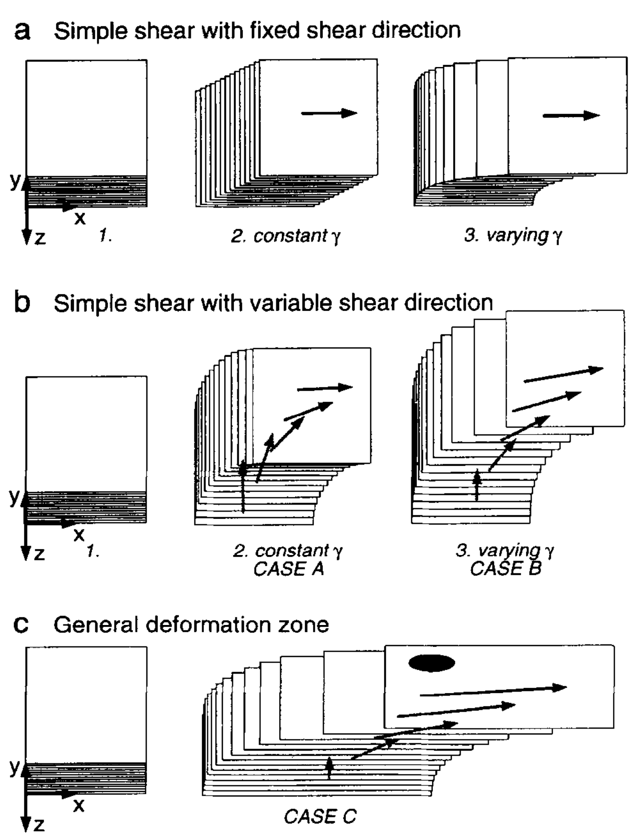

Fig. 7. Various types of simple shear in three dimensions. For all types the x–z co-ordinate plane is the shear plane, and x is the shear direction. (a) Simple shear *sensu stricto*, a plane strain. 1, Initial state. 2, Uniform shear (constant γ). 3, Heterogeneous simple shear. (b) Simple shear with a changing shear direction. 1, Initial state. 2, Uniform shear, considered as *Case A*. 2, Heterogeneous shear, our *Case B*. (c) General deformation zone, *Case C*, equivalent to *Case B* + zone-parallel stretching. See text for discussion.

Case B: heterogeneous shear with a changing shear direction. This is a more general type of shear zone (Fig. 7b) which may be regarded as a three-dimensional version of the characteristic shear zone (Fig. 6a), but with a changing shear azimuth. It might be said that such a deformation is bound to contravene the terms for principal surface continuity as it is a more general version of Case A. Mathematically, we have:

$$t_x = f_1(y),\ t_y = f_2(y),\ t_z = f_3(y). \tag{13}$$

Curl **t** has the components:

$$\begin{aligned}
[\text{curl } \mathbf{t}]_x &= \delta f_3(y)/\delta y \\
[\text{curl } \mathbf{t}]_y &= 0 \\
[\text{curl } \mathbf{t}]_z &= -\delta f_1(y)/\delta y.
\end{aligned} \tag{14}$$

This gives rise to the same expression for A as Case A (equation (12), confirming that XY surfaces cannot be defined for this type of shear zone.

It is clear from these two types of shear zones that it is the *changing direction* of shear across a zone, not the value or heterogeneity of the shear strain, which is the critical factor that defines the non-existence of continuous XY surfaces. The changing direction in three dimensions has a coiling torsional effect, leading to non-zero abnormality for the Z vector field. These types of shear zone would seem likely patterns of deformation for geological shear zones developed over time. Means (1984, 1995) considered two models of evolution for shear zones in rock, where Type 1 zones widen with time and Type 2 narrow with time. Now supposing there is a constant rate of shear affecting a progressively widening zone (as Type 1), but also that the shear direction changes progressively with time, the result would be a shear zone like our Case B, above. We reiterate that such a shear zone would not have continuous definable XY surfaces of finite strain, and this will have implications for related fabrics, as discussed later.

General deformation zones—Case C shear zone

The term 'general shear zone' has been used by Simpson and De Paor (1993) and others to mean a zone of combined simple shear and zone-parallel stretch (pure shear); or more generally, a combined simple shear, pure shear, dilation and rigid rotation. Despite the apparent generality, this is nevertheless an example of only two-dimensional deformation variation. We therefore prefer to use the term *general deformation zone* for our most general type of three-dimensional shear zone; termed Case C. This is a combination of pure shear and simple shear with changing orientation (Fig. 7c), equivalent to a Case B shear zone, with the addition of zone-parallel/perpendicular stretching, and perhaps including volume change. The stretching (pure-shear) component will not be parallel to the shear direction, except locally. This type of deformation need not be considered a zone in the sense of a restricted band, but could be of broader extent.

The vector components for Z strain axes for examples of this general deformation will not be presented. It is sufficient to establish that the vector curl components will be more complicated functions than those for the previous example (equations 11, 12 and 13), involving x, y and z terms, and therefore $A \neq 0$. Compared to the previous example, the stretch component will clearly have an effect on the twisting nature of principal axes, but will not alter the geometric property: that continuous XY surfaces will not be definable.

Three-dimensional strain refraction with viscosity variation

The variation of stress and strain across layers with viscosity contrast, and where layering is oblique to far-field stress and strain in either two or three dimensions, has been modelled by Treagus (1981, 1983, 1988). The general form of this kind of deformation variation has been shown to be characterized by a combination of a layer-parallel simple shear of fixed direction, but inversely proportional to viscosity, and a homogeneous layer-parallel pure shear. For three-dimensional strain refraction and variation, the two components do not share any of the same principal axes. Unlike the previous examples, where the simple-shear component had a progressive change in *direction* (Fig. 7b & c), here it is a combination of the different *amounts* of layer-parallel simple shear in different layers, and a constant pure-shear component (on different axes), that results in the three-dimensional twisting of principal strain axes (e.g. Treagus, 1988, fig. 5).

The sharp patterns of stress and strain refraction, modelled by Treagus (1988) for layered systems in oblique strain, is clearly a different scenario from the *smoothly* varying states of heterogeneous stress and strain, considered in the examples above. There geometrical variations without any material properties were considered. However, where viscosity can be considered to vary smoothly across a bed, such as in a graded horizon or across a zone, then the deformation variation might then be modelled as smoothly varying. An example of this is shown in Fig. 8, which adapts the results of sharp strain refraction from Treagus (1988, fig. 5) to an imaginary *graded unit* with smoothly changing viscosity, decreasing upwards. It is clear that a smoothly curving XY surface can exist for two-dimensional strain refraction (Fig. 8a), as already discussed for other examples (e.g. Fig. 6). The XY surface curves or refracts on an axis parallel to the intermediate strain axis, Y, and the X and Y axes are principal curvature trajectories, according to Dupin's theorem. However, for a 'bed' oblique to all three far-field (and thus local) strain axes, as shown in Fig. 8(b), a continuous XY surface cannot be defined. The supposed XY 'surface' can be represented schematically as a series of discontinuous strips, upwards, which have successive changes in intersection direction with bedding-parallel planes across the 'graded unit'. However, this is not a real surface, as would be established by attempting to draw a closed 'continuity loop'. It follows that a continuous refracting curved 'cleavage' should not be envisaged for this kind of three-dimensional deformation variation.

Ductile transpression zones

Transpression, a deformation which combines compression with transcurrent motion (Harland, 1971; Sanderson and Marchini, 1984), is traditionally treated as a homogeneous deformation across a discrete zone. The zone is usually represented as vertical and the motion vector as horizontal. Robin and Cruden (1994) drew attention to the boundary and mechanical problems of this ideal transpression model (Fig. 9a) and proposed a model of *ductile transpression* (Fig. 9b) which is a heterogeneous deformation. However, unlike heterogeneous simple shear zones, this transpressive zone has a uniform shear ('trans' component), and a heterogeneous pure shear ('press') which dies out to zero at the zone margin (Fig. 9b) (Robin and Cruden, 1994, fig. 6).

In their study, Robin and Cruden (1994) provide rare information on truly three-dimensionally heterogeneous deformation. They show strain-rate trajectories on block diagrams and stereograms, including diagrams showing 'foliation' traces (here the instantaneous plane of flattening) (e.g. Fig. 9c & d). Robin and Cruden (1994) do not appear to question whether the instantaneous XY surfaces ('foliation') are continuous. We will test this in one of their examples (their fig. 8). Taking the traces, together with stereographic data (Fig. 9c & d) (and also P.-Y. Robin, personal communication 1996), we have attempted to construct a 'continuity loop' in Fig. 9(e), but find that no closed loop can be drawn from the data. Therefore the traces on the block diagram in Fig. 9(c) are not traces of continuous instantaneous XY surfaces. Any supposed foliation likewise cannot be considered as a continuous surface.

This example has a strong transcurrent component

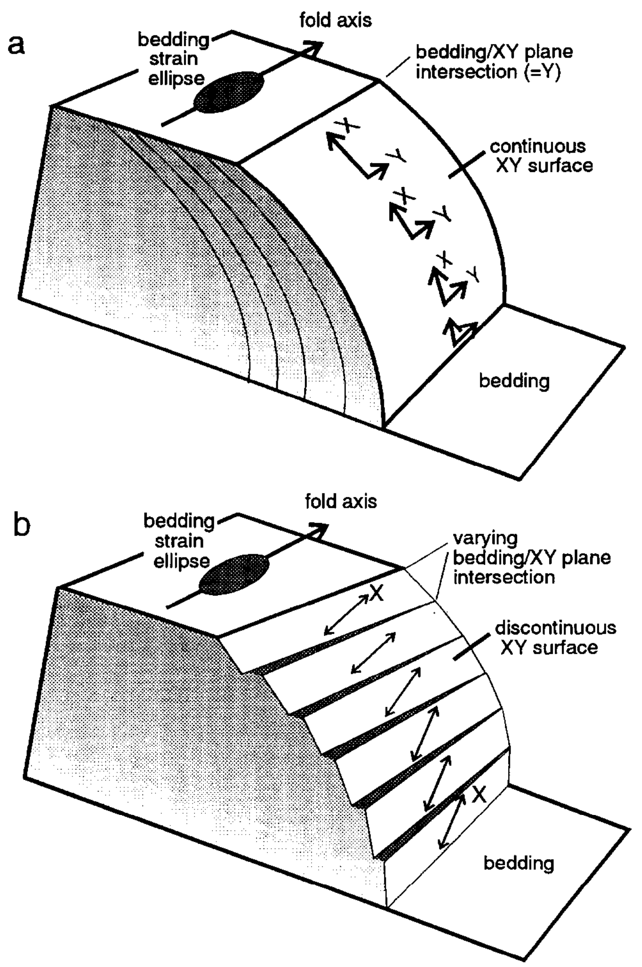

Fig. 8. Strain variation and XY plane orientations in a schematic graded bed with smoothly changing viscosity, decreasing upwards, based on results in Treagus (1988). (a) Two-dimensional strain refraction, Y parallel to bedding. (b) Three-dimensional strain refraction, with X, Y and Z axes all oblique to bedding, a discontinuous XY surface and a changing XY–bedding intersection.

($f=0.1$, where $f=$'press'/'trans'; Robin and Cruden, 1994), and the deformation is therefore less heterogeneous than if f were a higher value (as it is the 'press' component that is inhomogeneous; Robin and Cruden, 1994, fig. 6). The degree of discontinuity of principal surfaces might therefore be expected to be higher, where the 'press' component is greater. We consider the result in Fig. 9(e) sufficient to prove the case (for all f except 0 or ∞), that the ductile transpression model of Robin and Cruden (1994) will not generally give rise to continuous principal surfaces of stress or strain. This generality would appear to hold regardless of whether the 'trans' (shear) component is horizontal, as usually assumed for transpression, or oblique, as also considered in their analyses. If this type of transpression is considered as a more realistic model for geological deformations than the traditional homogeneous transpression of Sanderson and Marchini (1984), the implication is that the principal surfaces of stress and strain, and related structures and fabrics, should not be expected to be continuous surfaces.

Torsional deformations

Pure torsion was one of Mandl's examples of a deformation with discontinuous principal surfaces of stress (Mandl, 1987). We expanded on this in the Introduction, and demonstrated that torsion gives rise to cylindrical $\sigma_1\sigma_3$ surfaces, but that $\sigma_1\sigma_2$ and $\sigma_2\sigma_3$ surfaces (which might have been thought to be helicoidal) cannot be defined (Fig. 2b). In terms of strain, XY and YZ surfaces will not exist.

The geometry and possible geological environments of helices and helicoids have recently been investigated by Fowler (1996). He gives an example of helicoidal cleavage traces from the Lachlan Fold Belt in Australia, and suggests a torsional origin. The cleavage trace data provided by Fowler (1996, figs 7–9) do not include depth information, and so do not lend themselves to the 'continuity loop' test (Fig. 4; e.g. Fig. 9e). However, if the arcuate cleavage is indeed continuous, it would seem to disprove a torsional origin as torsion will not give rise to continuous XY surfaces. Alternatively, if the deformation *is* torsional, the cleavage would be expected to be discontinuous 'surfaces'; therefore the 'traces' shown by Fowler (1996) might be lines which are not traces of a real continuous cleavage surface in three dimensions.

Stress and strain fields around fracture tips

Since the time of the analysis of stresses around cracks by Griffith (1920) it has been recognized that flaws, cracks and through-going fractures (joints, faults) will give rise to complex heterogeneous stress fields in their vicinity (see Pollard and Segall, 1987). The nature of the regions of 'stress disturbance' which arise around flaws, or overlapping fractures, or as a result of specific motions on a fracture, is a vast field within fracture mechanics that we cannot attempt to cover here. Our point of interest is what the significance to fracture propagation might be of considering *heterogeneous stress fields* in three dimensions, compared to the traditional two-dimensional stress patterns derived by most of the theoretical or laboratory modelling. In these two-dimensional analyses, it is usual to see the fracture treated as a line and stress trajectories to be drawn in the plane of view, which is regarded as a principal plane (e.g. Thomas and Pollard, 1993). Yet if the fracture is a circular or elliptical crack propagating in two dimensions as a circle or ellipse (e.g. Willemse *et al.*, 1996), the full region of stress disturbance must be a heterogeneous stress field in three dimensions. The nature of this circular or elliptical annular region around a fracture tip-line will be a complex heterogeneous stress field, and must differ according to the type of fracture motion (Modes I, II or III; see Atkinson, 1987) and position with respect to the movement vector (e.g. frontal or lateral tip). We see no reason to suppose that the stress or strain vector fields for these regions will have 'zero abnormality' except, perhaps, locally.

We suggest that the general case along most fracture

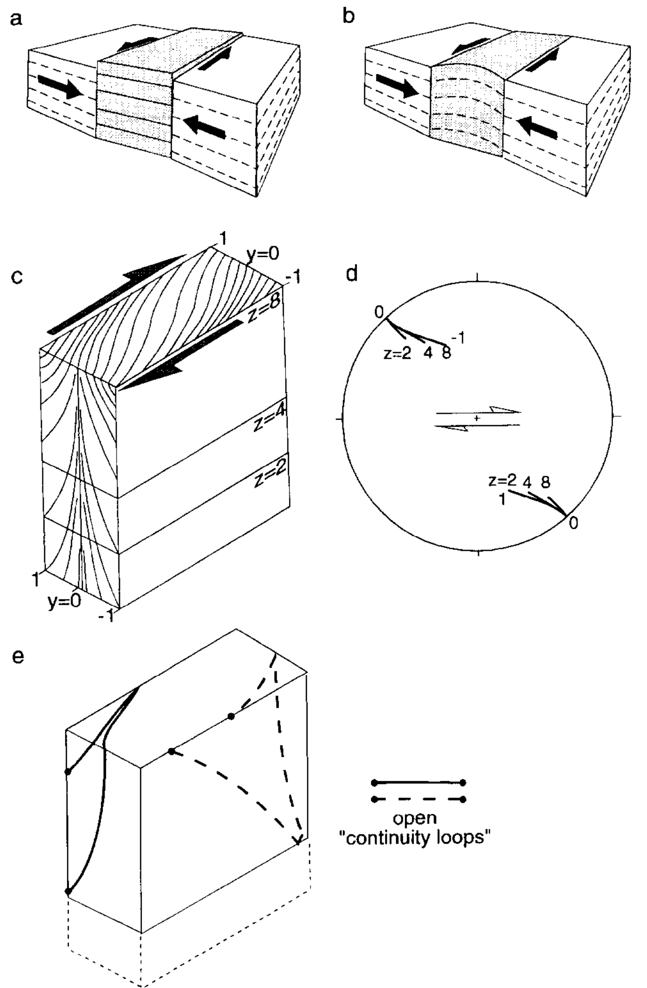

Fig. 9. The ductile transpression model. (a) Traditional (Sanderson and Marchini, 1984) transpression model and (b) the ductile transpression model, after Robin and Cruden (1994, fig. 1). (c) and (d) Example of ductile transpression, after Robin and Cruden (1994, fig. 8). (c) Block diagram (x, y, z co-ordinates) showing traces of 'foliation'. (d) Poles for the 'foliation' for y (0 to ± 1) and z (2, 4, 8). (e) Application of the continuity loop method for the block in (c) (taken from $z=2$ to $z=8$). Two different 'loops' are shown for different traces but both are open, showing that the 'foliation' cannot be a continuous surface.

tips will be a three-dimensionally heterogeneous stress field which does not possess the property of definable principal surfaces. If this is indeed the case, it is difficult to explain how any fracture might propagate as a continuous plane or surface.

DISCUSSION AND GEOLOGICAL APPLICATIONS

Many types of geological structures are traditionally described as closely related to principal directions of stress or strain. Brittle fracture-related structures such as joints, veins and dykes are usually considered as tensile fractures parallel to $\sigma_1\sigma_2$ surfaces, whereas faults are considered as oriented in the Mohr–Coulomb orientation (parallel to σ_2, acute to σ_1). Planar mineral fabrics, especially those arising from a single 'first' deformation, are generally equated to XY planes of strain, although not necessarily the total strain. We focus the discussion by considering examples of discontinuous brittle structures, examining some of the problems in assessing the continuity of supposed planar fabrics, and ending with a discussion of whether continuous linear traces in two dimensions might give a false impression of continuity in three dimensions.

Segmented faults and echelon cracks, veins and dykes

The starting point to our paper was Mandl's note (Mandl, 1987), itself apparently prompted by Segall and Pollard (1980) on *discontinuous fault zones*. He proposed that a segmented fault could be explained in terms of discontinuous Coulomb (shear) fractures (see Fig. 1b) arising under heterogeneous stress fields which do not possess continuous principal surfaces of stress. We have already noted in the Introduction (see earlier citations) the current interest in segmented faults and their importance in fault populations (see also Cowie *et al.*, 1996), but surprisingly little attention given to Mandl's hypothesis. The three-dimensional heterogeneity of stress around fracture tips, in addition to the kind of regional stress variation considered by Mandl (see Fig. 1), would seem to make discontinuous fracture propagation a likely phenomenon. Understanding when and why faults may be segmented, and how faults may link into large through-going faults or fault zones, have far-reaching implications to the movement of fluids in the Earth's crust and the locations of resulting economic resources.

Joint surfaces are commonly shown with echelon segments in a 'joint fringe' (Suppe, 1985, p. 173; Price and Cosgrove, 1990, p. 46) (Fig. 10a). It is understood that this is a joint edge effect, but it is less clear whether this is a characteristic of *all* lateral edges of joints, or a result of stress disturbance as a joint meets a barrier such as a different lithological unit, as might be deduced from Fig. 10(a). In the context of this paper, the question is whether joint fringes are indicative of stress systems which do not possess continuous $\sigma_1\sigma_2$ surfaces, or not. We leave this open.

Mineral veins are commonly seen in echelon arrays when viewed in cross-section, and have been shown to have many of the same properties as echelon cracks (Nicholson and Pollard, 1985; Nicholson and Ejiofor, 1987) (Fig. 10b). While sigmoidal vein arrays may appear as conjugate zones in cross-section (e.g. Ramsay and Huber, 1987, session 26), for which a sense of shear might be deduced, treating sigmoidal vein arrays as two-dimensional linear markers in shear zones may belie their complex three-dimensional geometry. In many cases, veins of traditional echelon sectional appearance can be shown to be complex three-dimensionally branching solid bodies (Fig. 10b), similar in geometry to other types of segmented fractures (cf. Figs 1b and 10a & c).

Igneous dykes show similar features to mineral veins

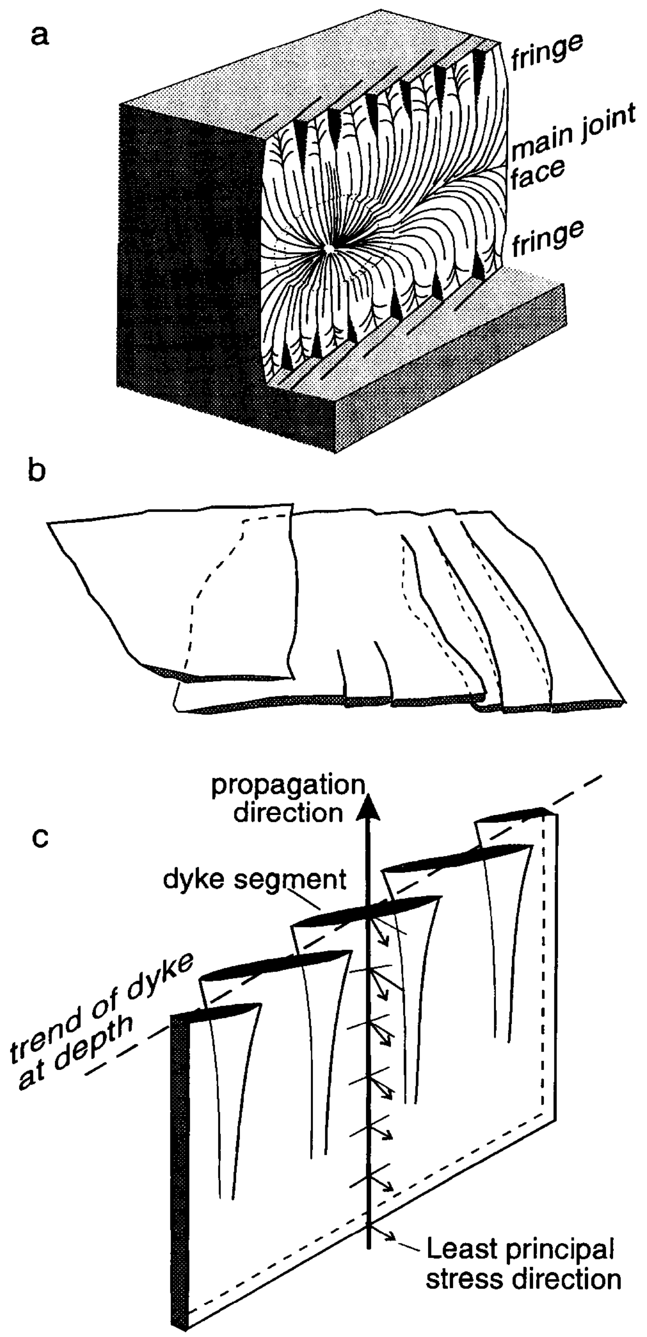

Fig. 10. Examples of discontinuous fractures and related structures. (a) Joint fringe structures, after Suppe (1985, fig. 6-4). (b) Three-dimensional form of veins, drawn from collapsing serial sections of Nicholson and Ejiofor (1987, fig. 3). Note changes from echelon segments at the front to linked veins at the back. (c) Schematic representation of echelon dykes, from Suppe (1985, fig. 7-7), attributed to Delaney and Pollard (1981).

(Delaney and Pollard, 1981; Nicholson and Pollard, 1985; Suppe, 1985, pp. 214–217) (Fig. 10c) because they, too, occupy tensile fractures. Sometimes all that is left of the original frontal array of dyke 'fingers' are angular wallrock protrusions or rafts in the dyke, marking remnant 'bridges' between the earlier dyke segments, or horn-like dyke margins (Nicholson and Pollard, 1985, fig. 4).

If fractures, whether tensile or shear, commonly initiate as discontinuous cracks, and become linked only by subsequent cross-fracture, this implies that the orientation of the through-going (linked) fracture is not a reliable indicator of the principal stress orientations at the time of fracture initiation. A sequential history of fracture initiation and linkage would appear, from evidence of veins and dykes, to be more easily preserved by tensile fractures than shear fractures. The latter might obliterate the early fracture patterns by successive fault movements, perhaps only preserving earlier discontinuous fractures as relict surfaces features such as ridges or grooves (Hancock and Barka, 1987).

Strain fabrics: foliations or not?

In earlier examples we considered a range of types of three-dimensionally heterogeneous strain, and found that for all but some special symmetric strain patterns, and plane strain, principal XY surfaces will not exist. What is the signifance of this for supposedly planar deformation fabrics such as cleavage, foliation and schistosity?

Pencil cleavage is sometimes described as a weak incipient cleavage which combines properties of a bedding-parallel fabric with a weak tectonic fabric, and is sometimes considered indicative of a prolate strain (see Reks and Gray, 1982; Ramsay and Huber, 1983, p. 185; Durney and Kisch, 1994). We offer another possible explanation: that it forms under deformation conditions where XY 'planes' are not continuous surfaces. This origin might be indicated by a changing (curving) orientation of the 'pencils' (indicating locally heterogeneous deformation), or the localization of the fabric at specific structural irregularities (e.g. in fold hinge regions or at structural terminations).

Slaty cleavage in true slates has demonstrable planar fissility (e.g. in roofing slates), and in these rocks it has been long established that the cleavage is subparallel to the XY planes of strain (e.g. Siddans, 1972; Wood, 1974). Cleavage surfaces in slates can often display remarkable continuity, a feature closely linked to their economic value as a building material. This probably reflects uniform lithology and close to homogeneous deformation: the opposite scenario from the heterogeneous three-dimensional deformation fields considered in this paper. Slate belts may thus provide examples where principal surfaces may be accurately described as *principal planes* over a reasonable scale of observation.

Where *cleavage transects folds* formed in the same deformation (see Treagus and Treagus, 1981, 1992, and references therein), we suggest this is evidence of a more general form of three-dimensional deformation. All three principal strain axes are likely to be oblique to layering, refracting through different lithologies. For this kind of three-dimensional deformation we expect, from previous research (see Fig. 8b), the following structural associations. (i) Cleavages do not have a constant cleavage–

bedding intersection around folds, or through successive beds. (ii) Stretching lineations are not perpendicular or parallel to fold axes and cleavage–bedding intersections. (iii) Cleavages transect folds. (iv) Cleavage refraction and fanning cannot be accurately represented by a continuous curving surface.

We have shown, with a range of examples of three-dimensionally heterogeneous deformation, that these will not generally have continuous XY surfaces. The question therefore arises: *are foliation surfaces for such deformations continuous planes or surfaces at all*? An accurate construction of the refracting or fanning fabric in block diagram form, and attempts to draw a continuity loop from cleavage traces (see Fig. 4), might allow us to distinguish true geometrically continuous fabric surfaces from 'surfaces' which might be considered only approximately continuous over a particular scale of observation. Foliations related to general heterogeneous deformation might be found not to be definable continuous surfaces on a regional scale, yet could appear to be planar and continuous at the thin section, hand specimen or, even, outcrop scale. So it is possible that even where deformation is macroscopically heterogeneous in three dimensions, and therefore does not have geometrically definable XY surfaces throughout the deformed region, the discontinuities will be small enough, or distributed as 'jumps', so that more-or-less continuous planar fabrics may be seen at the microscopic to mesoscopic scale.

Shear zones can develop a variety of complex structures and fabrics. We consider the special *S–C fabric* (Berthé *et al.*, 1979; Lister and Snoke, 1984; see also Passchier and Trouw, 1988, p. 113) which can develop in shear zones, particularly in crystalline rocks. The C-fabric (or C'-fabric) is of shear-band type, generally picked out by planar minerals, whereas the S-fabric is given by deformed grain shapes, and may be assumed to indicate the maximum extension (X) in cross-section. The sense of S–C fabrics, in sections across shear zones parallel to lineation, makes these useful shear-sense indicators. However, these fabrics are often only poorly defined and are far from true planar fabrics, except in completely mylonitized rock where S and C become indistinguishable. Lin and Williams (1992) provide evidence that S–C fabrics may anastomose in three dimensions. If it is generally the case that fabrics in shear zones are not true continuously planar fabrics, a point originally made by Berthé *et al.* (1979), but are locally anastomosing planes or simply collections of stretched grains (i.e. linear not planar), there becomes no real problem with the expected non-existence of continuous fabrics in more complex three-dimensional shear zones considered in the earlier examples. The main distinction between an anastomosing S–C fabric in a traditional two-dimensional simple shear zone, and a non-planar fabric in a three-dimensional shear zone with a changing direction of shear, would be the progressive change in shear direction and stretching lineation across the zone for the latter.

We have questioned a fundamental aspect of foliation surfaces by suggesting that these may not be continuous surfaces or planes in the rock at all! With the exception of true slates which cleave for considerable distances along the fabric, much of the observational data for cleavage, foliations and schistosities come from sections perpendicular to supposed cleavage surfaces (e.g. the wealth of examples in Borradaile *et al.*, 1982, chap. IV). Linear traces marked out by mineral alignment or pressure solution may first give the illusion of a continuous linear trace in a plane of section (e.g. thin section), and it may then be assumed without question that this is the trace of a continuous surface in three dimensions, even when the geometry may deny this possibility. We welcome comments on these controversial questions.

Changing from two to three dimensions: from traces to surfaces?

For many types of 'planar' geological structure, whether fractures or fabrics, the geological information is likely to be in the form of traces: on a map, in cross-section, from cut samples and thin sections. The procedure of mapping lithological contacts by linking adjacent exposures is sometimes applied to structures such as faults, dykes and even cleavage fabrics. The 'form mapping' of cleavage often reveals important regional swings of cleavage, whether presumed original or due to later deformation. However, we are unaware of any questioning about whether a constructed linear trace is *necessarily* indicative of a continuous surface.

Stratigraphic horizons and bedding planes within these, shown by outcrop or 'form surface' traces on the map, must generally have three-dimensional continuity. Stratum contours should, in theory, be constructable. Do we wrongly assume that this is also the case for *structures* and *fabrics*?

Fault lines on a map may generally be assumed to be traces of fault planes which go to some depth. Yet faults are commonly discontinuous, as discussed above, and so we must reject this as a general rule. *Cleavage traces* which are joined up as map trend-lines may not likewise represent continuous cleavage surfaces on the map scale. From successive map readings of strike/dip of first cleavage, it is easy to join up *any* series of strike bars into a 'trend-line', regardless of whether this is a trace of a real (continuous) surface. The most obvious example is where readings show changes in dip but no change in strike, leading to straight trend-lines simply representing discontinuous flaps (a 90°-rotated version of the flags in Fig. 1a). The important point is that continuous lines can easily be drawn on two-dimensional (sectional) surfaces, but this does not indicate that continuous surfaces can always be constructed in three dimensions.

We thus conclude on a note of caution. Sheets of paper which represent regions of geology or cross-sections, and photographs or thin sections, together constitute the main sources of data for geological structures. These may sometimes deceive the mind into simply projecting the two-dimensional data with 'tunnel vision' into three dimensions, so that all lines become continuous planes or surfaces where they may not be viable continuous surfaces at all.

CONCLUSIONS

(1) Three-dimensional variations of stress and strain, characterized by three families of curving stress or strain trajectories, will not generally possess definable curved principal surfaces of stress or strain.

(2) Continuity of principal surfaces can be tested mathematically by considering the abnormality of the vector field, or geometrically using the continuity loop.

(3) All two-dimensionally varying states of stress and strain have continuous principal surfaces. One family is planar; the other two are cylindrically curving, with lines of principal curvature parallel to principal axes.

(4) Geological deformations of the following types will not give rise to continuous principal surfaces of stress and strain, and so continuous fracture or XY fabric surfaces should not be expected.

(a) Simple shear with a changing direction of shear across a zone.

(b) All three principal stresses or strains oblique to geological layering, with competence contrast.

(c) Mutually oblique components of pure shear and simple shear, where one is heterogeneous. This is a generalized form of deformation, which includes (b), general deformation zones and ductile transpression.

(d) Torsional deformation.

(e) Stresses in regions of fracture tips, at least on the 'screw dislocation' parts of tips.

(5) Discontinuous brittle structures such as segmented faults, joint fringes, and echelon veins and dykes are likely to be indicative of no continuous principal surfaces of stress.

(6) Pencil cleavage, some forms of cleavage refraction and fanning, transecting cleavage patterns, and S–C shear-zone fabrics might all be the product of deformation with no continuous XY surfaces.

(7) Traces and form lines drawn on maps, cross-sections, serial sections and thin sections may not always represent the intersection of a real continuous 'form surface' in three dimensions.

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